Hydrostatic Pressure

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7 October 2014

BSEN 3010

Abstract:

The Hydrostatic Pressure lab used an Edibon Hydrostatic Pressure System to demonstrate how small changes in mass can cause changes in the amount of pressure required to counterbalance that mass with fluid pressure. The weights were added and then water was added until the system settled on the equivalence point on the apparatus. This was done with the tray of the Edibon System partially submerged and then totally submerged. When the tray was partially submerged, the graph created was a logarithmic graph with a y-intercept of 0 but when the tray was totally submerged the graph was linear. This is hypothesized to be because the amount of water needed to counteract the mass is less dependent on height, due to the fact that the water is pressing on the the curved surface of the tray and not the flat surface. The data was also plotted using center of pressure, height of the water, and the standard deviation. For the flat surface, the standard deviation was low but very erratic but with the totally submerged surface, the standard deviation increased slightly at a very linear rate. All standard deviations in the experiment ranged from 1.3 to 3.3.

Introduction:

A static fluid is any fluid that is at rest (Cengel and Cimbala, 2014). Fluids at rest are of great interest because they are all held in by an outer boundary. This boundary must be able to withstand the pressure caused by the fluid pressing against it so determining the total force of the fluid on the boundary is critical to the stability of the boundary. Because these boundaries are keeping the fluid from moving, they are considered hydrostatic systems and in hydrostatic systems there is only normal stress, or pressure, and no shear stress that is present in moving bodies of liquid or gas. (Cengel and Cimbala, 2014)

Hydrostatic pressure systems can be seen on a regular basis in everyday life. Such examples are river dams, water towers, and storage tanks. Each of these examples is an example of a vital static water system. River dams are used to prevent flooding of cities as well as the construction of manmade lakes. Water towers house back up water supplies for many different towns and cities all over the country. Storage tanks often house dangerous chemicals used by companies in production of vital goods we use every day.

One very important example is the use of hydrostatic systems as a tool for livestock production. Many farms rely on ponds and storage tanks for a water supply, especially during the drier summer months. For instance, some farms utilize storage ponds for water storage. (Water Storage 1960) These ponds must have dams to hold in water that would otherwise continue running downstream and dissipating into the ground where it would no longer be available for drinking. The maintenance of the dam is very important because if the dam were to bust the entire pond could be lost. For this reason it is very important to know how much pressure the dam needs to be able to withstand. Depending on the size and depth of the pond, some dams may need to be more fortified than others. For instance, some dams are acceptable when covered only with grass and shrubs (Water Storage 1960) while others must be fortified with concrete. Understanding how to calculate the total pressure applied to a hydrostatic system boundary will allow engineers and farmers to know exactly how they need to build their dams for their farms to have the best success.

Objectives:

During this experiment, students used an Edibon Hydrostatic Pressure System to see how water pressure affected a flat surface on a small scale. Equations were given and then proved so students can see how the small-scale model can be used to model a large-scale application.

Methods and Materials:

This lab utilized the Edibon Hydrostatic Pressure system. Weights were added to the balance arm after the mass was taken and then water was added to the tank to balance out the added weight. The masses of the weights were recorded, as was the depth of the water on the graduated tray. These were then plugged in to equations given with the lab and the results were graphed using Microsoft Excel. The equations given were

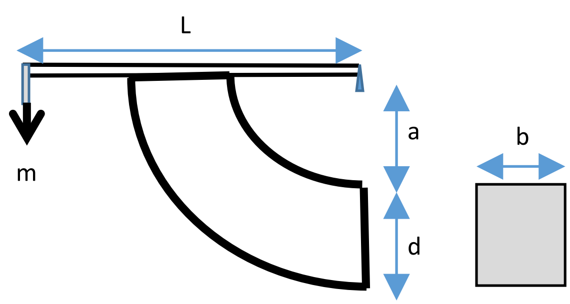
(1) $m = \frac{\rho b}{2L}\left(a + d - \frac{h}{3}\right) * y^2$

(2) $h_p = a + d - \frac{h}{3}$

and

(3) $F = \frac{1}{2}\rho * g * h^2 * b$

They corresponded with the figure given with the lab:



a =100 mm, d =100 mm, b =70 mm and L =285 mm

The equations were given but they could have been derived. The derivation of Equation (1) starts with the volume equation

$$V = bLh_p$$

where V is volume, b is the width of the tank, L is the length of the balance arm, and h_p is the center of pressure on the plane surface. To derive Equation (1), Equations (2) and (3) must be found.

Equation (2) is the depth from the balance arm to the top of the tray, *a*, plus the depth of the tray, *d*, minus the distance from the bottom of the tray to the centroid of the pressure triangle that is 1/3 of the height of the triangle.

Equation (3) starts with the generic force equation

$$F = p_c * A$$

where F is force, p_c is the pressure at the centroid, and A is the surface area of the tray. The pressure at the centroid is then broken down to

$$p_c = \rho * g * h_c * A$$

where ρ is the density of water, 1000 kg/m^3, g is the force of gravity, 9.81 m/s^2, and h_c is the height of the center of pressure on the face of the tray. Because the tray is a square surface, h_c is simply $\frac{1}{2}h$. Plugging back into the equation $F = p_c * A$ gives Equation (3)

$$F = \frac{1}{2}\rho * g * h^2 * b$$

To finish the derivation of Equation (1), the moment around the pivot point of the balance arm must be considered. The sum of the moments must be equal to zero when the mass and the tray are in equilibrium. Here F is the force applied to the tray. It is negative because it is clockwise while the moment created by the masses is counterclockwise. The moment created by the masses is *W*L* where *W* is the weight of the masses added to the end of the balance arm.

$$\sum M_x = 0 = -F * h_p + W * L$$

This equation can be rearranged to be

$$F * h_p = W * L$$

Which can then be broken down to

$$m * g * L = F * h_p$$

where m is the mass of the weights and g is the acceleration due to gravity, 9.81 m/s^2. This equation is rearranged into

$$m * g = \frac{F * h_p}{L}$$

This can be substituted into

$$m * g = \frac{F * h_p}{L} = \frac{1}{2L} * \rho * g * y^2 * b\left(a + b + \frac{y}{3}\right)$$

Gravity can be cancelled out on each side of the equation to finally render Equation (1).

$$m = \frac{\rho b}{2L}\left(a + b - \frac{y}{3}\right)y^2$$

Results and Discussion:

After recording the data obtained from the experiment, the equations and data were put into and Excel spreadsheet and the following graphs were generated.

Figure 1 shows the relationship between the mass added to the end of the balance arm and the y-value, or height of the water on the tray. As the amount of weight increased, the relative amount of water decreased. This is why the graph requires a polynomial trendline. Something else that can be noted here is the fact that there is no y-intercept for the equation. This is because the Edibon Hydrostatic Pressure System was zeroed out when there was no weight on the balance arm. It is not surprising that the graph has a logarithmic shape because in Equation (1) it was determined that the y value on the end of the equation is squared therefore it will not have to increase as fast as the mass value in order to keep the system balanced.

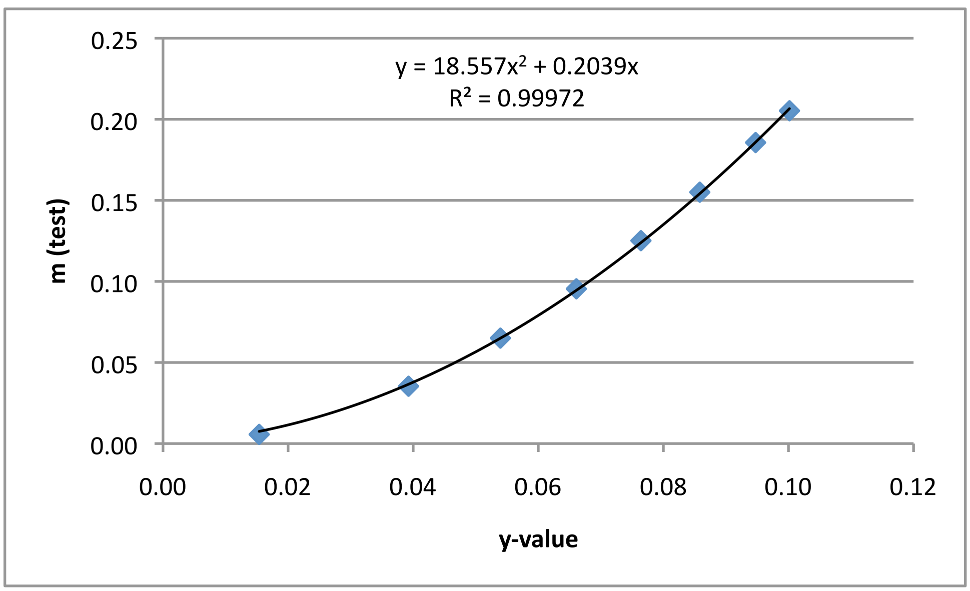


Figure 1: Mass of the weights vs. the theoretical height of the water for a partially submerged tray

Figure 2 shows the graph of the y-value calculated using Excel and the height of the water on the plane surface that was observed in lab. The linear relationship, along with an almost 1:1 ratio in the trendline, shows that the y values that were calculated were very close to accurate.

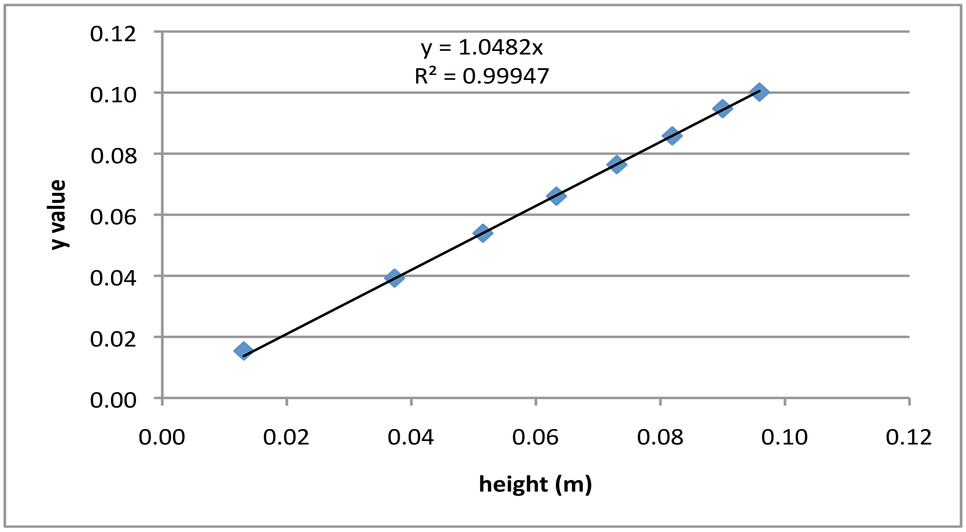


Figure 2: Plot of the calculated y-value vs. the recorded heights during the experiment for the partly submerged tray

Figure 3 shows the relationship between the height of the center of pressure and the height of the water required to bring the apparatus to equilibrium. The line generated by the data is exactly linear. The line has a negative slope because the Height of the Center of Pressure is measured from the balance arm. Therefore when the water level rises, the distance between the top of the water level and the balance arm decreases. The graph also has a y-intercept value listed. This is because the balance was zeroed with the balance arm exactly 0.2 meters above the water level.

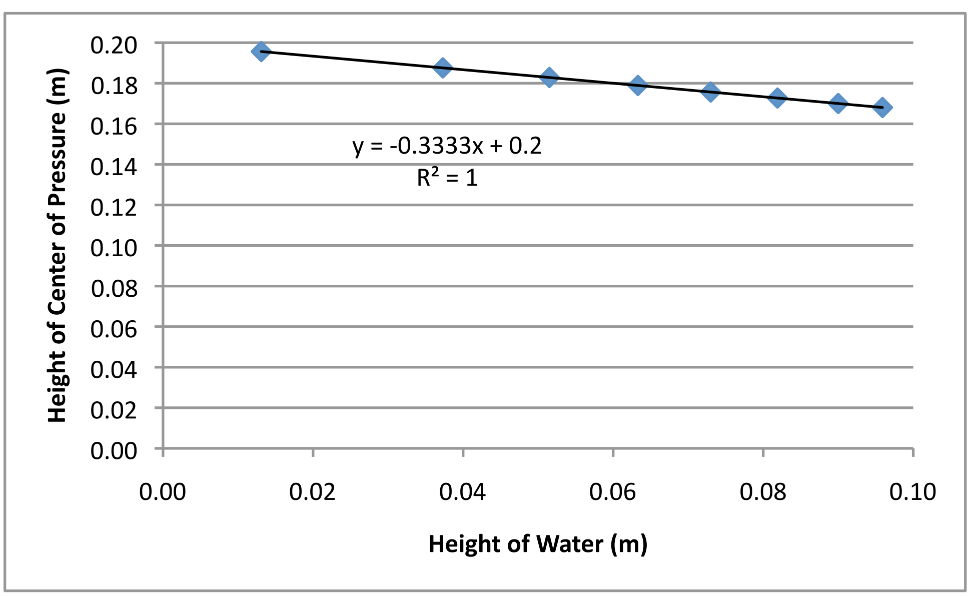


Figure 3: Height of Center of Pressure vs. Height of Water for Partially Submerged Tray

Figure 4 illustrates the relationship between the force created by the water pressing on the partially submerged tray in relation to the height of the water. Here, force is measured using Equation (3) and the height of the water was recorded during the experiment. This graph is exponential because the height of water is squared in Equation (3). The coefficient of the constant is very large because all of the force of the water is acting directly on the plane surface of the tray. It can also be seen that the y-intercept for this graph is 0. This is expected because, if there is no water in contact with the tray, there is no force acting on the tray.

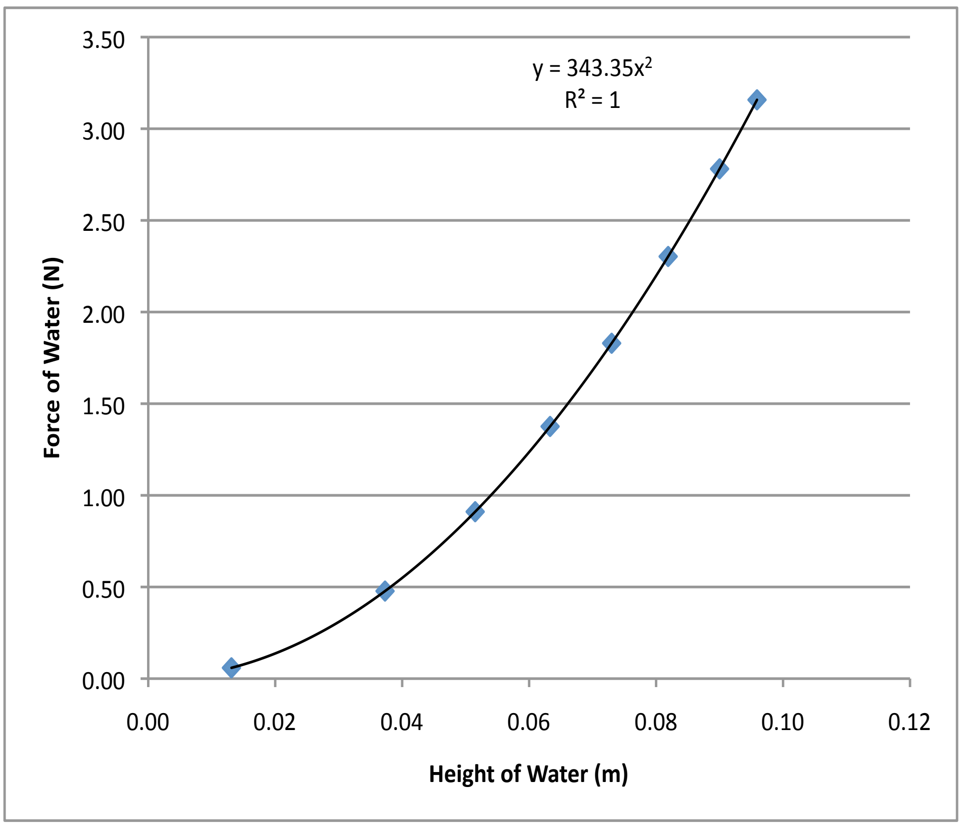


Figure 4: Force of the water acting on the tray vs. the height of the water for a partially submerged surface

Figure 5 shows the standard deviation of our recorded heights. The graph is very scattered which tends to indicate that some values were correct, those with lower standard deviations and some values, those with the higher standard deviations, were less accurate. Even though some of the standard deviations were scattered around the graph, they are all fairly low, meaning that the results collected were similar to what they should have been.

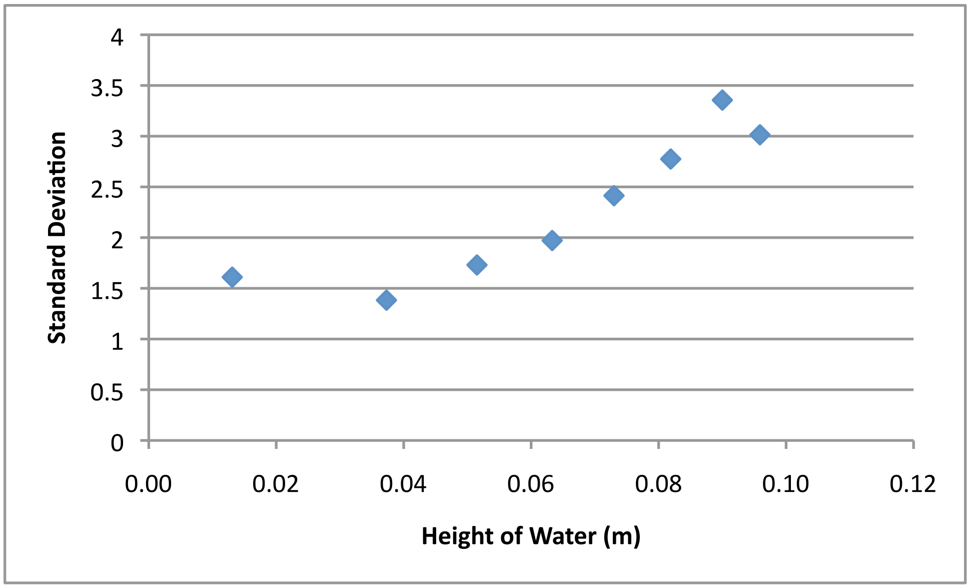


Figure 5: The standard deviation of the height recorded versus the ideal height calculated for a partially submerged tray.

Figure 6 is the graph of the mass of the weight used in the fully submerged half of the lab. Here the relationship is linear, unlike above in the partially submerged tray. This is because the amount of water needed to counter balance the mass is more constant when there is more water pushing on the surface. One can also notice the y-intercept in the equation for the line. This is present because the bottom of the water is no longer starting at the base of the tray, it is starting above the bend in the tray.

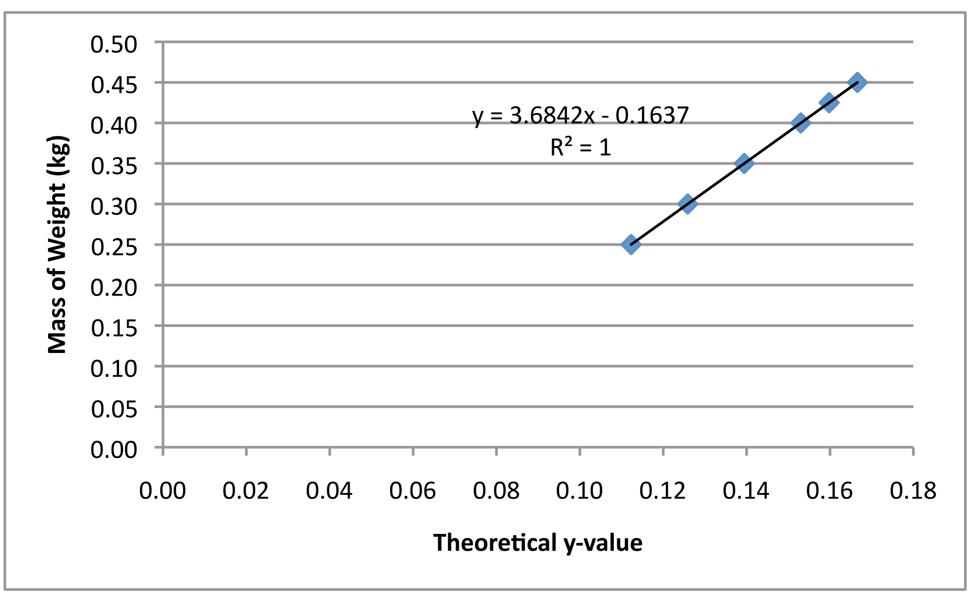


Figure 6: Mass of weights vs. the theoretical y-value of a submerged tray

Figure 7 below is the graph of the theoretical y-value plotted against the water heights calculated in the lab for a totally submerged surface. Here the relationship is not as linear as the same graph for the partially submerged surface, indicating an error on the part of those doing the experiment. This could have come because of a misread of equipment, or because of a slight calibration error. It may have also occurred because it is harder to get exact measurement with a totally submerged tray.

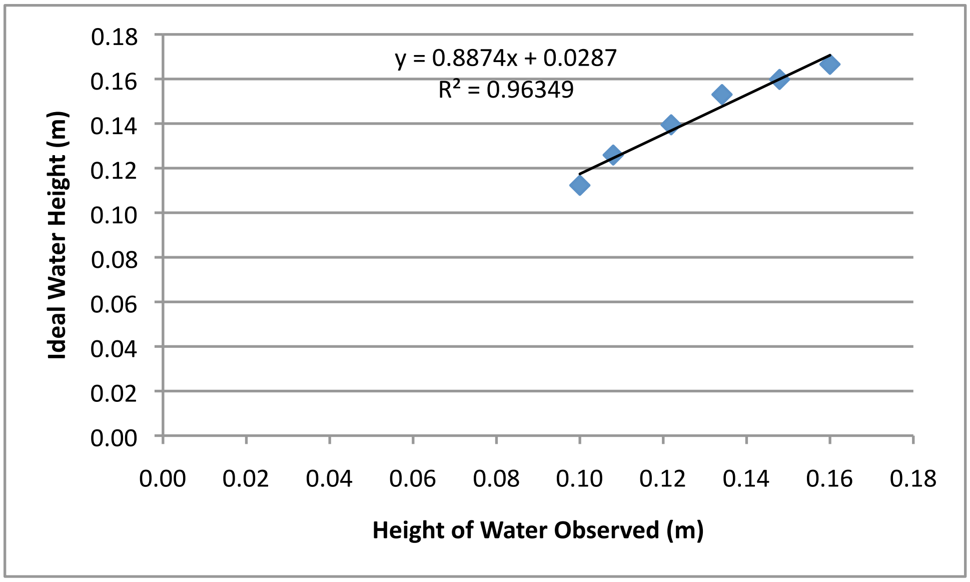


Figure 7: Ideal water height vs. the observed water height for a totally submerged tray

Figure 8 is the relationship between the center of pressure versus the observed height of the water in the tank. The equation for the trendline again has a y-intercept to account for the original distance between the balance arm and the top of the water. As with the partially submerged tray, the slope is negative because as the water level is raised, the distance from the balance arm is decreasing.

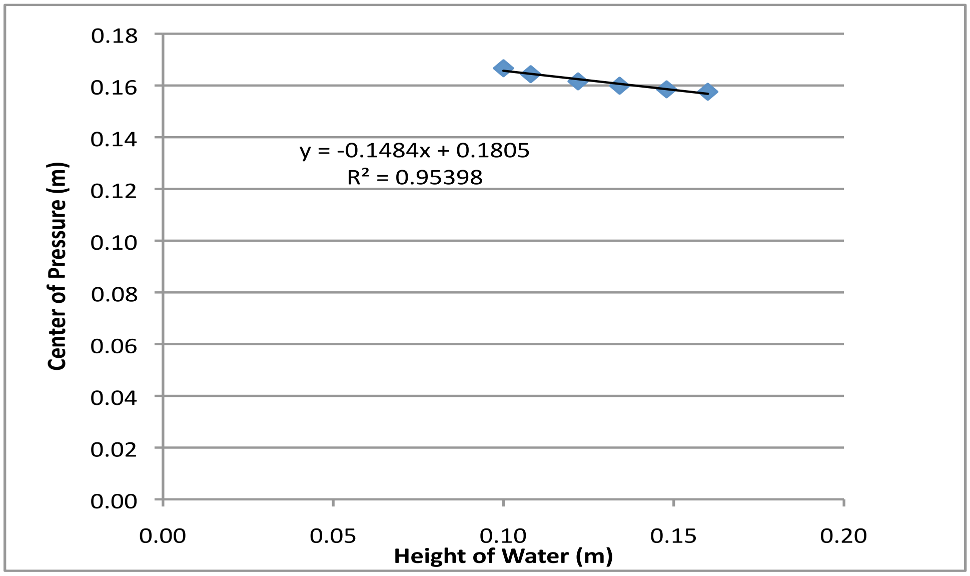


Figure 8: Center of pressure vs. the recorded height of water for a fully submerged tray

Figure 9 represents the force acting on the tray versus the amount of water in the tank. Here the force is calculated by using the equation:

$$F = \rho * g * b\left(h - \frac{d}{2}\right)d$$

Here it can be seen that the pressure on the block is increasing at a slower rate than it was on the partially submerged block. This is because the water is now acting down on the tray, as well as against the plane face. Some of the force that would normally be taken to push directly against the face is now dissipated in other direction.

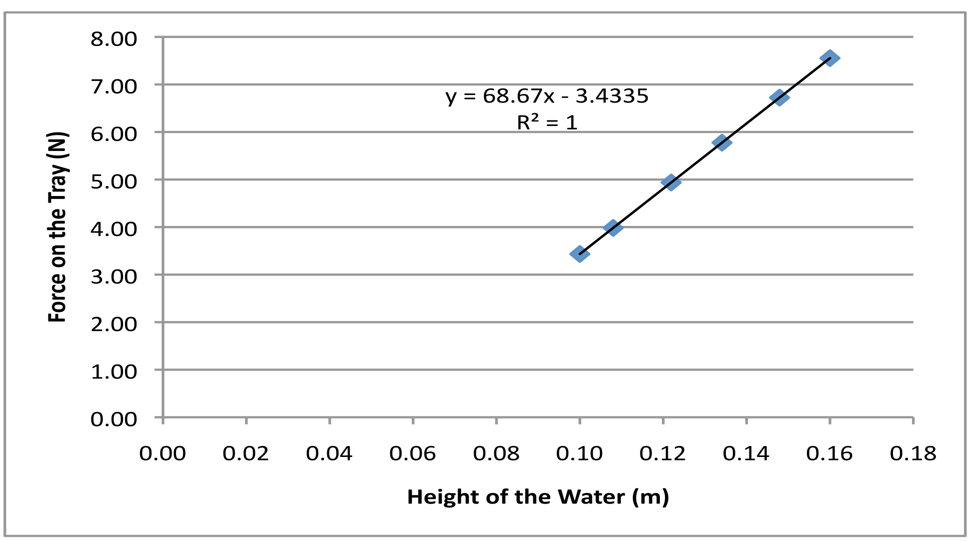


Figure 9: Force of the water on the submerged tray versus the height of the water

Figure 10 shows the standard deviation of the heights recorded during lab from the ideal heights calculated in Excel. This graph has an excellent linear shape with a positive slope. This leads to the assumption that there is more error involved when calculating a submerged surface, and the error increases as the surface gets deeper in the water.

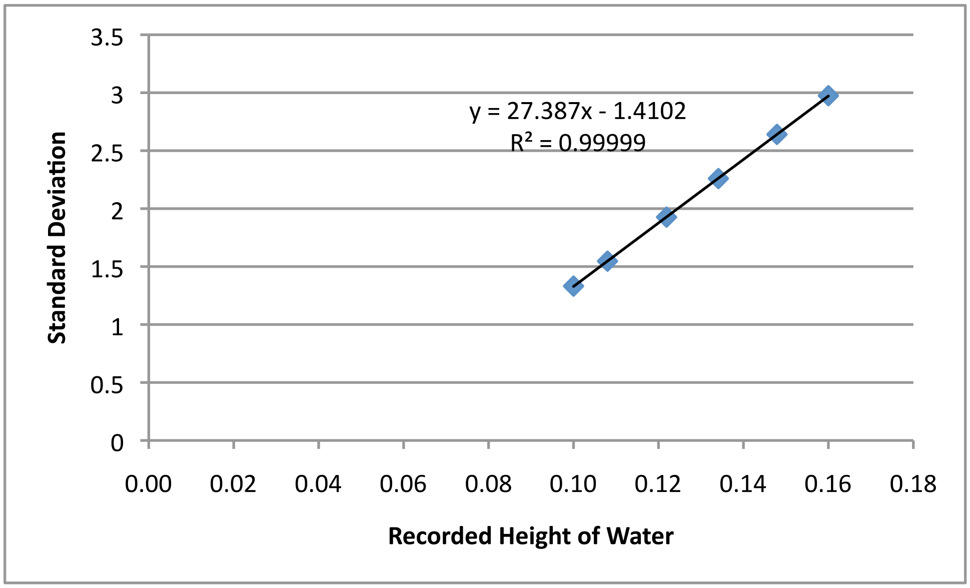


Figure 10: Standard Deviation of the heights recorded from the heights calculated for a fully submerged tray

<u>Conclusion</u>:

This lab provided the opportunity to see, on a small scale, how depth can affect the pressure on a plane and submerged surface by hanging masses on a balance arm and then counteracting that force with the force of water against the surface. It also provided a chance to prove the equations given during the lab were in fact true. This leads to the proof that these equations can be used in real world examples on a much larger scale. After completing the lab, the data proved that as depth increases, the force applied to the center of pressure increases. It was also evident that the force from a hydrostatic system is much greater on a flat surface than it is a curved surface.

References:

Cengel, Y. and J. Cimbala. *Fluid Mechanics: Fundamentals and Applications.* 3rd ed. New York, New York: McGraw-Hill, 2014. 89. Print.

"Water Storage Requirements of Farm Reservoirs." *Transactions of the ASAE* 3.1 (1960): 63-64. Web.